Exercise 291

For the following exercises, solve the logarithmic equation exactly, if possible.

$$\ln x + \ln(x-2) = \ln 4$$

Solution

Combine the logarithms on the left side.

$$\ln x(x-2) = \ln 4$$

For both sides to be equal, the arguments must be equal.

$$x(x-2) = 4$$

Expand the left side.

$$x^2 - 2x = 4$$

Bring all terms to one side.

$$x^2 - 2x - 4 = 0 \tag{1}$$

Observe the coefficient of x is -2. Divide this number by 2 and then square it.

$$-2 \quad \rightarrow \quad \frac{-2}{2} = -1 \quad \rightarrow \quad \left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$$

This is what needs to be added to both sides of equation (1) in order to complete the square.

$$x^2 - 2x + 1 - 4 = 0 + 1$$

$$(x-1)^2 - 4 = 1$$

$$(x-1)^2 = 5$$

Take the square root of both sides.

$$\sqrt{(x-1)^2} = \sqrt{5}$$

Since there's an even power (2) under an even root (2) and the result is an odd power (1), the result needs an absolute value sign.

$$|x-1| = \sqrt{5}$$

Place $\pm$ on the right side to remove the absolute value sign.

$$x - 1 = \pm\sqrt{5}$$

Solve for x.

$$x = 1 \pm \sqrt{5}$$

Now plug these solutions into the original equation. The logarithm of a negative number is undefined, so the solution $x = 1 - \sqrt{5} \approx -1.23$ is discarded. Therefore,

$$x = 1 + \sqrt{5}.$$